5.2 Using Intercepts

Essential Question: How can you identify and use intercepts in linear relationships?

Explore Identifying Intercepts

Miners are exploring 90 feet underground. The miners ascend in an elevator at a constant rate over a period of 3 minutes until they reach the surface. In the coordinate grid, the horizontal axis represents the time in minutes from when the miners start ascending, and the vertical axis represents the miners’ elevation relative to the surface in feet.

A What point represents the miners’ elevation at the beginning of the ascent? Plot this point.

B What point represents the miners’ elevation at the end of the ascent? Plot this point.

C Connect the points with a line segment.

D What is the point where the graph crosses the y-axis? the x-axis?

Reflect

1. Discussion The point where the graph intersects the y-axis represents the beginning of the miners’ ascent. Will the point where a graph intersects the y-axis always be the lowest point on a linear graph? Explain.
**Explain 1**  
**Determining Intercepts of Linear Equations**

The graph in the Explore intersected the axes at (0, –90) and (3, 0).

The y-intercept of a graph is the y-coordinate of the point where the graph intersects the y-axis. The x-coordinate of this point is always 0. The y-intercept of the graph in the Explore is –90.

The x-intercept of a graph is the x-coordinate of the point where the graph intersects the x-axis. The y-coordinate of this point is always 0. The x-intercept of the graph in the Explore is 3.

**Example 1**  
**Find the x- and y-intercepts.**

**A**  
3x – 2y = 6  

To find the x-intercept, replace y with 0 and solve for x.

3x – 2(0) = 6
3x = 6
x = 2

The x-intercept is 2.

To find the y-intercept, replace x with 0 and solve for y.

3(0) – 2y = 6
−2y = 6
y = −3

The y-intercept is −3.

**B**  
−5x + 6y = 60

To find the x-intercept, replace y with ______ and solve for x.

−5x + 6(______) = 60

−5x = 60
x = ______

The x-intercept is ______

To find the y-intercept, replace ______ with 0 and solve for ______.

−5(______) + 6y = 60

6y = 60
y = ______

The y-intercept is ______

**Reflect**

2. If the point (5, 0) is on a graph, is (5, 0) the y-intercept of the graph? Explain.

**Your Turn**

**Find the x- and y-intercepts.**

3. 8x + 7y = 28

4. −6x − 8y = 24
**Interpreting Intercepts of Linear Equations**

You can use intercepts to interpret a situation that is modeled by a linear function.

**Example 2** Find and interpret the \(x\)- and \(y\)-intercepts for each situation.

A The Sandia Peak Tramway in Albuquerque, New Mexico, travels a distance of about 4500 meters to the top of Sandia Peak. Its speed is 300 meters per minute. The function \(f(x) = 4500 - 300x\) gives the tram's distance in meters from the top of the peak after \(x\) minutes.

To find the \(x\)-intercept, replace \(f(x)\) with 0 and solve for \(x\).

\[
0 = 4500 - 300x \\
x = 15
\]

It takes 15 minutes to reach the peak.

To find the \(y\)-intercept, replace \(x\) with 0 and find \(f(0)\).

\[
f(0) = 4500 - 300(0) = 4500
\]

The distance from the peak when it starts is 4500 m.

B A hot air balloon is 750 meters above the ground and begins to descend at a constant rate of 25 meters per minute. The function \(f(x) = 750 - 25x\) represents the height of the hot air balloon after \(x\) minutes.

To find the \(x\)-intercept, replace \(f(x)\) with 0 and solve \(x\).

\[
0 = 750 - 25x \\
x = \boxed{30}
\]

It takes 30 minutes to reach the ground.

To find the \(y\)-intercept, replace \(x\) with 0 and find \(f(0)\).

\[
f(0) = 750 - 25(30) = 750
\]

The height above ground when it starts is 750 m.

**Reflect**

5. **Critique Reasoning** A classmate says that the graph shows the path of the tram. Do you agree?

6. **Your Turn** The temperature in an experiment is increased at a constant rate over a period of time until the temperature reaches 0 °C. The equation \(y = \frac{5}{2}x - 70\) gives the temperature \(y\) in degrees Celsius \(x\) hours after the experiment begins. Find and interpret the \(x\)- and \(y\)-intercepts.
Explain 3  Graphing Linear Equations Using Intercepts

You can use the $x$- and $y$-intercepts to graph a linear equation.

**Example 3**  Use intercepts to graph the line described by each equation.

(A) \( \frac{1}{2}y = 3 - \frac{3}{4}x \)

Write the equation in standard form. \( \frac{3}{4}x + \frac{1}{2}y = 3 \)

Find the intercepts.

- $x$-intercept: \( \frac{3}{4}x + \frac{1}{2}(0) = 3 \)
  \[ \frac{3}{4}x = 3 \]
  \[ x = 4 \]

- $y$-intercept: \( \frac{3}{4}(0) + \frac{1}{2}y = 3 \)
  \[ \frac{1}{2}y = 3 \]
  \[ y = 6 \]

Graph the line by plotting the points (4, 0) and (0, 6) and drawing a line through them.

(B) \( 18y = 12x + 108 \)

Write the equation in standard form. \( 18y = 12x + 108 \)

Find the intercepts.

- $x$-intercept: \( -12x + 18(0) = 108 \)
  \[ -12x = 108 \]
  \[ x = -9 \]

- $y$-intercept: \( -12 + 18y = 108 \)
  \[ 18y = 120 \]
  \[ y = 6.67 \]

Graph the line by plotting the points (-9, 0) and (0, 6.67) and drawing a line through them.

**Your Turn**

7. Use intercepts to graph \( 3y = -5x - 30 \).
**Elaborate**

8. A line intersects the $y$-axis at the point $(a, b)$. Is $a = 0$? Is $b = 0$? Explain.

9. What does a negative $y$-intercept mean for a real-world application?

10. **Essential Question Check-in** How can you find the $x$-intercept of the graph of a linear equation using the equation? How is using the graph of a linear equation to find the intercepts like using the equation?

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**Evaluate: Homework and Practice**

Identify and interpret the intercepts for each situation, plot the points on the graph, and connect the points with a line segment.

1. An electronics manufacturer has 140 capacitors, and the same number of capacitors is needed for each circuit board made. The manufacturer uses the capacitors to make 35 circuit boards.

2. A dolphin is 42 feet underwater and ascends at a constant rate for 14 seconds until it reaches the surface.
Find the $x$- and $y$-intercepts.

3. $2x - 3y = -6$  
4. $-4x - 5y = 40$

5. $8x + 4y = -56$  
6. $-9x + 6y = 72$

7. $\frac{3}{5}x + \frac{1}{2}y = 30$  
8. $-\frac{3}{4}x + \frac{5}{6}y = 15$

Interpret the intercepts for each situation. Use the intercepts to graph the function.

9. **Biology**  
   A lake was stocked with 350 trout. Each year, the population decreases by 14. The population of trout in the lake after $x$ years is represented by the function $f(x) = 350 - 14x$.

10. The air temperature is $-6^\circ C$ at sunrise and rises $3^\circ C$ every hour for several hours. The air temperature after $x$ hours is represented by the function $f(x) = 3x - 6$. 
11. The number of brake pads needed for a car is 4, and a manufacturing plant has 480 brake pads. The number of brake pads remaining after brake pads have been installed on \( x \) cars is \( f(x) = 480 - 4x \).

12. Connor is running a 10-kilometer cross country race. He runs 1 kilometer every 4 minutes. Connor’s distance from the finish line after \( x \) minutes is represented by the function \( f(x) = 10 - \frac{1}{4}x \).

Use intercepts to graph the line described by each equation.

13. \(-6y = -4x + 24\)  

14. \(9y = 3x + 18\)  

15. \(y = \frac{1}{5}x + 2\)  

16. \(-3y = 7x - 21\)
17. \( \frac{3}{2}x = -4y - 12 \)

18. \( \frac{2}{3}y = 2 - \frac{1}{2}x \)

19. Kim owes her friend $245 and plans to pay $35 per week. Write an equation of the function that shows the amount Kim owes after \( x \) weeks. Then find and interpret the intercepts of the function.

20. **Explain the Error** Arlo incorrectly found the \( x \)-intercept of \( 9x + 12y = 144 \).

   His work is shown.
   
   \[
   9x + 12y = 144 \\
   9(0) + 12y = 144 \\
   12y = 144 \\
   y = 12 \text{ The } x \text{-intercept is 12.}
   \]

   Explain Arlo's error.

21. Determine whether each point could represent an \( x \)-intercept, \( y \)-intercept, both, or neither.

   A. (0, 5)  
   B. (0, 0)  
   C. (–7, 0)  
   D. (3, –4)  
   E. (19, 0)

22. A bank employee notices an abandoned checking account with a balance of $360. The bank charges an $8 monthly fee for the account.

   a. Write and graph the equation that gives the balance \( f(x) \) in dollars as a function of the number of months, \( x \).

   b. Find and interpret the \( x \)- and \( y \)-intercepts.
23. Kathryn is walking on a treadmill at a constant pace for 30 minutes. She has programmed the treadmill for a 2-mile walk. The display counts backward to show the distance remaining.

a. Write and graph the equation that gives the distance \( f(x) \) left in miles as a function of the number \( x \) of minutes she has been walking.

b. Find and interpret the \( x \)- and \( y \)-intercepts.

![Graph showing distance remaining vs. time]

**H.O.T. Focus on Higher Order Thinking**

24. **Represent Real-World Problems** Write a real-world problem that could be modeled by a linear function whose \( x \)-intercept is 6 and whose \( y \)-intercept is 60.

25. **Draw Conclusions** For any linear equation \( Ax + By = C \), what are the intercepts in terms of \( A \), \( B \), and \( C \)?

26. **Multiple Representations** Find the intercepts of \( 3x + 40y = 1200 \). Explain how to use the intercepts to determine appropriate scales for the graph and then create a graph.
Lesson Performance Task

A sail on a boat is in the shape of a right triangle. If the sail is superimposed on a coordinate plane, the point where the horizontal and vertical sides meet is (0, 0) and the sail is above and to the right of (0, 0). The equation of the line that represents the sail’s hypotenuse in feet is $10x + 4y = 240$.

a. Find and interpret the intercepts of the line and use them to graph the line. Then use the triangle formed by the $x$-axis, $y$-axis, and the line described by the above equation to find the area of the sail.

b. Now find the area of a sail whose hypotenuse is described by the equation $Ax + By = C$, where $A$, $B$, and $C$ are all positive.